

2001 Paper 5 Question 9

Semantics of Programming Languages

- (a) The integer expressions E of a programming language are given by

$$E ::= n \mid X \mid -E \mid E + E$$

where n ranges over integer constants and X ranges over identifiers. Explain the principle of *structural induction* for proving that some property $\Phi(E)$ holds for all integer expressions E . [5 marks]

- (b) Taking *states* to be finite partial functions mapping identifiers to integer constants, define a relation

$$E, s \Downarrow n$$

giving the result n (if any) of evaluating integer expression E in state s .

[7 marks]

- (c) Use structural induction to prove that if $E, s \Downarrow n_1$ and $E, s \Downarrow n_2$ both hold, then $n_1 = n_2$. [Hint: Consider the property $\Phi(E)$ given by $\forall s, n_1, n_2 ((E, s \Downarrow n_1) \ \& \ (E, s \Downarrow n_2) \Rightarrow n_1 = n_2)$.] [7 marks]

- (d) What property of the pair E, s ensures that there is some n for which $E, s \Downarrow n$ holds? [1 mark]