Semantics of Programming Languages

(a) The integer expressions $E$ of a programming language are given by

$$E ::= n \mid X \mid -E \mid E + E$$

where $n$ ranges over integer constants and $X$ ranges over identifiers. Explain the principle of structural induction for proving that some property $\Phi(E)$ holds for all integer expressions $E$. [5 marks]

(b) Taking states to be finite partial functions mapping identifiers to integer constants, define a relation

$$E, s \downarrow n$$

giving the result $n$ (if any) of evaluating integer expression $E$ in state $s$. [7 marks]

(c) Use structural induction to prove that if $E, s \downarrow n_1$ and $E, s \downarrow n_2$ both hold, then $n_1 = n_2$. [Hint: Consider the property $\Phi(E)$ given by $\forall s, n_1, n_2 ((E, s \downarrow n_1) \& (E, s \downarrow n_2) \Rightarrow n_1 = n_2)$.] [7 marks]

(d) What property of the pair $E, s$ ensures that there is some $n$ for which $E, s \downarrow n$ holds? [1 mark]