2001 Paper 4 Question 1

Continuous Mathematics

The complex form of the Fourier series is:

$$f(x) = \sum_{k=-\infty}^{+\infty} c_k e^{i2\pi kx}$$

where c_k is a complex number and $c_k = c^*_{-k}$.

(*a*) Prove that the complex coefficient, c_k, encodes the amplitude and phase coefficients, A_k and ϕ_k, in the alternative form:

$$f(x) = \sum_{k=0}^{+\infty} A_k \cos(2\pi kx - \phi_k)$$

[10 marks]

(*b*) What is special about the case $k = 0$? [2 marks]

(*c*) Explain how the coefficients, c_k, of the Fourier series of the periodic function, $f(x)$:

$$f(x) = f(x+T), \forall x$$

can be obtained from the Fourier transform, $F_L(\nu)$, of the related function, $f_L(x)$:

$$f_L(x) = \begin{cases} f(x), & -\frac{T}{2} \leqslant x < \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$$

[8 marks]