Regular Languages and Finite Automata

(a) Suppose that $L$ is a language over a finite alphabet $\Sigma$ with the property that for each number $\ell \geq 1$ there is some string $w$ in $L$ with $\text{length}(w) \geq \ell$ such that no matter how $w$ is split up into three pieces $w = u_1 v u_2$ with $\text{length}(u_1 v) \leq \ell$ and $\text{length}(v) \geq 1$, there is some $n \geq 0$ for which $u_1 v^n u_2$ is not in $L$. Prove that $L$ cannot be a regular language. [12 marks]

(b) State, with justification, whether each of the following languages over $\Sigma = \{a, b\}$ is regular.

(i) $L_1 = \{ww | w \in \Sigma^*\}$. [5 marks]

(ii) $L_2 = \{wvw | v, w \in \Sigma^*\}$. [3 marks]