2001 Paper 1 Question 2

Discrete Mathematics

(*a*) Prove the fundamental theorem of arithmetic, that any natural number can be expressed as a product of powers of primes and that such an expression is unique up to the order of the primes. [4 marks]

(*b*) Given a natural number n, let $d(n)$ be the number of divisors of n (including 1 and n).

If $p_1, p_2, \ldots, p_k$ are distinct primes, prove that

$$d({p_1}^{\alpha_1}{p_2}^{\alpha_2}\ldots{p_k}^{\alpha_k}) = \prod_{i=1}^{k}(\alpha_i + 1).$$ [3 marks]

(*c*) What is the smallest number with 36 factors? [3 marks]