

2001 Paper 12 Question 13

Complexity Theory

(a) Give a precise definition of the complexity class NP and of NP -completeness.
[2 marks each]

(b) For any natural number k , the problem **k-colourability** is defined as the following decision problem.

Given a graph $G = (V, E)$, is there a mapping $\chi : V \rightarrow \{c_1, \dots, c_k\}$ such that if $(u, v) \in E$, then $\chi(u) \neq \chi(v)$?

(i) Explain why, for each k , the problem **k-colourability** is in the class NP.
[2 marks]

(ii) For what values of k is the problem **k-colourability** decidable in polynomial time? Why?
[2 marks]

(iii) For which values of k is the problem **k-colourability** NP-complete? Give a brief indication how this might be proved.
[2 marks]

(c) The company Fon-X runs a mobile 'phone service. It has 2000 'phone masts stationed across the country. The frequency spectrum assigned to the company is split into 20 bands. Each mast is to be assigned a frequency band in such a way that masts within 50 miles of each other do not share the same frequency band.

(i) What is the relationship between this problem and **k-colourability**?
[2 marks]

(ii) What can you say about the complexity of the problem Fon-X is trying to solve?
[3 marks]

(d) Fon-X solved its frequency assignment problem by an exhaustive search algorithm, which took a week to run. The company has just doubled in size through a merger. It intends to repeat the frequency assignment on 4000 masts, setting aside two weeks for the task. As a consultant, write a short note to the company explaining what you think of the idea, and suggesting any alternatives you think might be better.
[5 marks]