

2000 Paper 6 Question 11

Logic and Proof

For each of the given pairs of terms, give a most general unifier or indicate why none exists. (Here x, y, z are variables while a, b are constant symbols.)

$$\begin{aligned}h(x, y, x) & \text{ and } h(y, z, u) \\h(x, y, z) & \text{ and } h(f(y), z, x) \\h(x, y, b) & \text{ and } h(a, x, y) \\h(x, y, z) & \text{ and } h(g(y, y), g(z, z), g(u, u))\end{aligned}$$

[4 marks]

A standard unification algorithm takes a pair of terms t_1 and t_2 and returns a substitution θ such that $t_1\theta = t_2\theta$. Show how this algorithm can be used to find the unifier of several ($n > 2$) terms t_1, t_2, \dots, t_n : a substitution θ such that $t_1\theta = t_2\theta = \dots = t_n\theta$. Indicate how the unifier is constructed from the unifiers of $n - 1$ pairs of terms. (Assume that all required unifiers exist and ignore the question of whether the unifiers are most general.) [6 marks]

Prove using resolution the formula

$$(\forall x [P(x) \leftrightarrow (Q(x) \wedge \neg Q(f(x)))]]) \rightarrow \exists y \neg P(y)$$

or indicate why this formula is not a theorem.

[10 marks]