Semantics of Programming Languages

The *pish* shell features commands which return an integer exit status. Write

\[(\ast) \quad C, s \downarrow n, s'\]

to indicate that command \(C\) started in state \(s\) terminates with exit status \(n\) and final state \(s'\). An exit status of zero indicates *normal* termination and a non-zero status indicates *abnormal* termination. The different forms of *pish* command are:

- skip (immediate normal termination)
- \(C ; C'\) (execute \(C\) and if it terminates normally continue with \(C'\)),
- if \(B\) then \(C\) else \(C'\) (execute \(C\) or \(C'\) according to the value of the boolean expression \(B\))
- if \(B\) return \(n\) (return exit status \(n\) if \(B\) is true, otherwise terminate normally)
- \(C\) handle \(n\) with \(C'\) (execute \(C\) and if it terminates with status \(n\), execute \(C'\))

In all cases the final exit status and state is that produced by the last command executed. Define the structural operational semantics of these commands by giving an inductive definition of \(\ast\). You may assume there is a relation of the form \(B, s \downarrow b\) (where \(b \in \{\text{true}, \text{false}\}\)) which defines the value of each boolean expression \(B\) in state \(s\). [7 marks]

Write \(C \cong C'\) to mean that for all \(s, n\) and \(s'\), it is the case that \(C, s \downarrow n, s'\) holds if and only if \(C', s \downarrow n, s'\) does. Show how to construct *pish* commands \(C_1\), \(C_2\) and \(C_3\) from \(C\), \(C'\), \(B\) and \(\text{true}\) just using the “if – return –” and “– handle – with –” constructs so that

(a) \(C_1 \cong \text{skip}\) [2 marks]
(b) \(C_2 \cong C ; C'\) [4 marks]
(c) \(C_3 \cong \text{if } B \text{ then } C \text{ else } C'\) [7 marks]

Justify your answer in each case.