Logic and Proof

Given a propositional formula, we wish to test whether it is a tautology and, if it is not, to compute an interpretation that makes it false. Two techniques for doing this are the sequent calculus and ordered-binary decision diagrams. Give a brief outline of these techniques, applying both of them to the formulae

\[(A \to B) \to (B \to A) \text{ and } (A \lor B) \to (\neg B \to A)\]

[7 + 7 marks]

It is proposed to replace the usual sequent calculus rule for disjunction on the left by this rule:

\[
\Gamma, A \Rightarrow \Delta \quad \Gamma, B \Rightarrow \Delta, A \\
\Gamma, A \lor B \Rightarrow \Delta
\]

Is this rule sound? Justify your answer. [3 marks]

Give an example to show that using this rule instead of the usual one makes some proofs shorter. [3 marks]