Regular Languages and Finite Automata

For each kind of regular expression over an alphabet Σ, define the language $L(r)$ of strings matching a regular expression $r$ of that kind. [4 marks]

Define the language $L(M)$ accepted by a deterministic finite automaton $M$. [2 marks]

Prove that for every deterministic finite automaton $M$ with alphabet of input symbols Σ it is possible to construct a regular expression $r$ over Σ satisfying $L(r) = L(M)$. [10 marks]

Illustrate your proof by constructing such an $r$ for the deterministic finite automaton with state set {0, 1, 2}, alphabet of input symbols {a, b}, initial state 0, accepting states 1 and 2, and next-state function

$(0, a) \mapsto 2, \quad (1, a) \mapsto 1, \quad (2, a) \mapsto 0,$
$(0, b) \mapsto 1, \quad (1, b) \mapsto 0, \quad (2, b) \mapsto 2.$

[4 marks]