Discrete Mathematics

What does it mean for a partial order to be well founded? [3 marks]

Given two well founded, partially ordered sets \((A, \leq_A)\) and \((B, \leq_B)\), define the lexicographic order on \(A \times B\) and show that it is well founded. [5 marks]

Two elements \(x\) and \(y\) of a partially ordered set are said to be separated if for all \(k \geq 1\) there is a sequence of elements \(z_1, z_2, \ldots z_k\) with \(x < z_1 < z_2 < \cdots < z_k < y\).

Give an example of a well founded, partially ordered set that contains infinitely many pairs of separated elements. [5 marks]

Prove that no well founded, partially ordered set has every pair of elements separated. [7 marks]