

2000 Paper 1 Question 7

Discrete Mathematics

What does it mean for a partial order to be *well founded*? [3 marks]

Given two well founded, partially ordered sets (A, \leq_A) and (B, \leq_B) , define the *lexicographic order* on $A \times B$ and show that it is well founded. [5 marks]

Two elements x and y of a partially ordered set are said to be *separated* if for all $k \geq 1$ there is a sequence of elements z_1, z_2, \dots, z_k with $x < z_1 < z_2 < \dots < z_k < y$.

Give an example of a well founded, partially ordered set that contains infinitely many pairs of separated elements. [5 marks]

Prove that no well founded, partially ordered set has every pair of elements separated. [7 marks]