Complexity Theory

State the hierarchy theorems for time and space. [4 marks]

A linear time reduction from a language \( L_1 \) to \( L_2 \) is a reduction that can be computed by a deterministic Turing machine in time \( O(n) \).

A class of languages \( C \) is closed under linear time reductions if whenever \( L_2 \in C \) and \( L_1 \) is linear-time reducible to \( L_2 \), then \( L_1 \in C \).

For each of the following complexity classes (a) to (d), say

- whether it is closed under linear time reductions
- whether it contains problems that are complete under linear time reductions

Give full justification for your answers.

(a) \( \text{DSPACE}(n^2) \) [4 marks]

(b) \( \text{L} \) [4 marks]

(c) \( \text{P} \) [4 marks]

(d) \( \text{NP} \) [4 marks]