

# 2000 Paper 12 Question 11

## Introduction to Functional Programming

Consider the following definitions of the functionals `foldl` and `foldr`:

```
fun foldl f e []      = e
  | foldl f e (h::t) = foldl f (f(h,e)) t;

fun foldr f e []      = e
  | foldr f e (h::t) = f(h, foldr f e t);
```

What is the type of `foldl`? [2 marks]

What is the type of the expression `foldr op/?` [2 marks]

For *each* of the following functions, write an ML definition using one of the functionals `foldl` or `foldr`.

- (a) `product: (real list) -> real`, which given a list of real numbers gives their product.
- (b) `exists: ('a -> bool) -> ('a list) -> bool`, which given a predicate  $p$  and a list  $l$  determines whether there is any element of  $l$  satisfying  $p$ .
- (c) `length: ('a list) -> int` which determines the length of a list.

[9 marks]

Prove, by induction on lists, that for all lists of integers  $l$ , the following identity is true:

$$\text{foldl } \text{op} + 0 \ l = \text{foldr } \text{op} + 0 \ l$$

[7 marks]