Computation Theory

Let \( \mathbb{N} \) be the natural numbers \( \{0, 1, 2 \ldots \} \).

What is meant by each of the following statements?

- The subset \( S \subseteq \mathbb{N} \) is recursive.
- The subset \( S \subseteq \mathbb{N} \) is recursively enumerable.

How would you extend the definition of recursive enumeration to sets of computable functions?

A sequence of natural numbers is a total function \( s : \mathbb{N} \to \mathbb{N} \). The sequence is recursive if and only if \( s \) is computable.

A finite sequence \( \sigma \) of natural numbers is specified by a pair \((l, x)\), where \( l \in \mathbb{N} \) is the number of elements, and \( x : [1, l] \to \mathbb{N} \) is a function that defines those elements. The case \( l = 0 \) defines the null sequence.

In each of the following cases, establish whether the set defined is recursively enumerable:

(a) the set of all recursive subsets of \( \mathbb{N} \)
(b) the set of all recursive sequences of natural numbers
(c) the set of all finite sequences of natural numbers