2000 Paper 11 Question 9

Computation Theory

Let $\mathbb{N}$ be the natural numbers $\{0, 1, 2 \ldots\}$.

What is meant by each of the following statements?

- The subset $S \subseteq \mathbb{N}$ is *recursive*.
- The subset $S \subseteq \mathbb{N}$ is *recursively enumerable*.

[5 marks]

How would you extend the definition of *recursive enumeration* to sets of computable functions? [3 marks]

A sequence of natural numbers is a total function $s : \mathbb{N} \to \mathbb{N}$. The sequence is *recursive* if and only if s is computable.

A finite sequence σ of natural numbers is specified by a pair (l, x), where $l \in \mathbb{N}$ is the number of elements, and $x : [1, l] \to \mathbb{N}$ is a function that defines those elements. The case $l = 0$ defines the null sequence.

In each of the following cases, establish whether the set defined is recursively enumerable:

(a) the set of all recursive subsets of $\mathbb{N}$ [5 marks]

(b) the set of all recursive sequences of natural numbers [2 marks]

(c) the set of all finite sequences of natural numbers [5 marks]