Numerical Analysis II

With reference to solution of the differential equation $y' = f(x, y)$, explain the conventional notation $x_n$, $y(x_n)$, $y_n$, and $f_n$. [3 marks]

Explain the terms local error, global error, and order of a method. [3 marks]

Milne’s method uses the multistep formulae

$$y_{n+1} = y_{n-3} + \frac{4h}{3} (2f_n - f_{n-1} + 2f_{n-2})$$

$$y_{n+1} = y_{n-1} + \frac{h}{3} (\tilde{f}_{n+1} + 4f_n + f_{n-1})$$

which each have local error $O(h^5)$. Outline the general technique for deriving multistep formulae. What is the meaning of the term $\tilde{f}_{n+1}$? Suggest a suitable starting procedure and explain how the Milne formulae are used. [8 marks]

Let $y(x_0) = 0$, $h = 0.3$ and $f(x, y) = 3y/x - 2$. Suppose the following values of $f_n$ have been generated by the starting procedure: 1.3, 2.1, 3.4 for $n = 1, 2, 3$. Calculate the first required value of $\tilde{f}_{n+1}$. [3 marks]

Contrast Milne’s method with a comparable one step method, commenting particularly on stability, efficiency and step size considerations. [3 marks]