Optimising Compilers

Consider a first-order call-by-need language with identifiers \( x_i \) and expressions \( e_i \) (which can be of type \texttt{int} only) and function names \( A_i \) (built-in) and \( G \) (a single, possibly recursive, user-defined function of the form \( G(x_1, \ldots, x_k) = e \)) whose arguments and results are of type \texttt{int}.

Describe the basic concepts of strictness analysis. You should explain what space of abstract values you would use to model strictness properties of a function of \( k \) arguments, and give the abstract strictness values for \texttt{cond} and \texttt{plus} (respectively the ternary conditional function and the binary addition function). State how one can determine that “\( f \) is strict in its \( i^{th} \) argument” in terms of your abstract value for a function \( f \) and how the abstract strictness value for \( G \) is obtained. [10 marks]

As an alternative method of deriving strictness properties, it is proposed to use an effect-like system instead. Suppose \( f \) is a function of \( k \) arguments and \( S \) is a subset of \( \{1, \ldots, k\} \). In such a system judgements on functions \( f \) are of the form

\[
\Gamma \vdash f : \texttt{int}^k \xrightarrow{S} \texttt{int}
\]

where \( \Gamma \) is a set of type assumptions on variables \( x \). The above judgement is defined to be valid if, whenever \( f \) is applied and all argument expressions in argument positions \( S \) fail to terminate, then the call to \( f \) fails to terminate (such \( f \) are often called \( S \)-jointly strict). In the following \( t \) ranges over type and effect forms \( \texttt{int}^k \xrightarrow{S} \texttt{int} \).

Give an inference rule (here called (SUB)) for judgements of the form \( \Gamma \vdash f : t \) which captures the idea that “if \( f \) is \( S \)-jointly strict in argument positions \( S \), then a call to it fails to terminate when applied to argument expressions which fail to terminate for a larger set of argument positions”. [3 marks]

Give a suitable set of assumptions of the form

\[
\Gamma_0 = \{ \texttt{plus} : t_1, \texttt{plus} : t_2, \texttt{cond} : t_3, \texttt{cond} : t_4 \}
\]

which together with the (SUB) rule above enable one to deduce exactly the valid strictness judgements \( \Gamma_0 \vdash f : t \) when \( f \) is \texttt{plus} or \texttt{cond}. [Hint: there are two \( t_i \) for both \texttt{plus} and \texttt{cond}.] [4 marks]

Give conditions on \( t \) in the judgement \( \Gamma \vdash f : t \) which enables the claim “\( f \) is strict in its \( i^{th} \) argument” to be made. [3 marks]