Logic and Proof

How do Prolog clauses differ from the clauses used by general-purpose resolution theorem provers? [2 marks]

Describe the series of resolutions that is performed by a Prolog interpreter when it is supplied with a program and a set of clauses. To illustrate your answer, explain how the following program executes when presented with the goal $\leftarrow Q(f(f(f(a))))$:

$$
\begin{align*}
Q(a) & \leftarrow \\
Q(f(a)) & \leftarrow \\
Q(f(f(x))) & \leftarrow Q(f(x)), Q(x)
\end{align*}
$$

[7 marks]

Considering the program and the goal $\leftarrow Q(f(f(f(f(a))))$ now as a set of clauses, derive the empty clause using general resolution. (Full credit requires finding the shortest derivation.) [6 marks]

Consider the set of clauses consisting of the program given above and the goal $\leftarrow Q(f(f(f(f(\ldots)))))$. Let $p(n)$ be the number of steps executed by a Prolog interpreter when given those clauses. Let $r(n)$ be the minimum number of steps required to derive the empty clause from those clauses using general resolution. Compare the approximate growth rates of $p(n)$ and $r(n)$ as $n$ increases, and explain any difference you find. [5 marks]