Suppose that $L$ is a language over the alphabet $\{0, 1\}$. Let $L'$ consist of all strings $u'$ over $\{0, 1\}$ with the property that there is some string $u \in L$ with the same length as $u'$ and differing from $u'$ in at most one position in the string. Show that if $L$ is regular, then so is $L'$. [Hint: if $Q$ is the set of states of some finite automaton accepting $L$, construct a non-deterministic automaton accepting $L'$ with states $Q \times \{0, 1\}$, where the second component counts how many differences have been seen so far.]

If a deterministic finite automaton $M$ accepts any string at all, it accepts one whose length is less than the number of states in $M$. Explain why.

State Kleene’s theorem about regular expressions and deterministic finite automata.

Describe how to decide for any given regular expression whether or not there is a string that matches it.