1999 Paper 10 Question 12

Numerical Analysis I

Define *absolute error*, *relative error* and *machine epsilon* ε_m. Although ε_m is defined in terms of absolute error, why is it useful as a measurement of relative error?

[4 marks]

For a floating-point implementation with $p = 4$, $\beta = 10$, explain the *round to even* method of rounding using the half-way cases 7.3125, 7.3175 as examples.

Now consider $p = 4$, $\beta = 2$. What is the value of ε_m? What should each of the following numbers be rounded to, using *round to even*?

1.0101 1.1100 1.0011 1.1001 [6 marks]

Suppose $\cos 6$ is calculated by summing the series

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

Estimate the value of the term with largest magnitude. Assuming this term can be computed with a relative error of 10^{-7}, what is the *absolute error* in computing this term? Hence, assuming $\cos 6 \simeq 1$, estimate the *relative error* in the computed value of $\cos 6$ to the nearest power of 10. [5 marks]

What are *guard digits*? How would you compute $\sqrt{x^2 - 2^{24}}$ if there was a danger that x^2 might overflow? If both x and powers of 2 are exactly represented, and guard digits are used, estimate the relative error in the result if $\varepsilon_m = 10^{-7}$.

[5 marks]