

## 1999 Paper 10 Question 10

### Mathematics for Computation Theory

Let  $A, B$  be sets. Define:

- (a) the *Cartesian product*  $(A \times B)$
- (b) the set of relations  $R$  between  $A$  and  $B$
- (c) the identity relation  $\Delta_A$  on the set  $A$

[3 marks]

Suppose  $S, T$  are relations between  $A$  and  $B$ , and between  $B$  and  $C$ , respectively. Define the inverse relation  $S^{-1}$  and the product relation  $S \circ T$ . Prove that  $(S \circ T)^{-1} = T^{-1} \circ S^{-1}$ . [4 marks]

Let  $f$  be a relation between  $A$  and  $B$ , and  $R$  be a relation on  $A$ . Characterise the following conditions in terms of the algebra of relations:

- (a)  $f$  is a partial function
- (b)  $R$  is reflexive
- (c)  $R$  is symmetric
- (d)  $R$  is transitive

[5 marks]

If condition (a) holds, let  $Q = f \circ f^{-1}$ . Which of conditions (b–d) must  $Q$  satisfy? In what circumstances is  $Q$  an equivalence relation? [8 marks]