

1998 Paper 5 Question 9

Foundations of Functional Programming

State the connection between the β -equality relation ($=_\beta$) and the β -reduction relation on λ -terms. Prove that $=_\beta$ is non-trivial, in the sense that there exist λ -terms M and N such that $M \neq_\beta N$. [4 marks]

Compare the call-by-name and call-by-value reduction strategies, giving examples to illustrate that

- (a) sometimes the call-by-name strategy gives fewer reductions than the call-by-value strategy, and *vice versa*;
- (b) the call-by-name strategy terminates when the normal form exists, whereas the call-by-value strategy need not.

[6 marks]

Given the λ -term $(\lambda x.xI)(\lambda y.(\lambda z.zzzz)(yt))$ where I is $\lambda u.u$, display reduction paths arising from the call-by-name and call-by-value reduction strategies. Also, find the reduction path which consists of the fewest reduction steps and comment on your answer. [10 marks]