Numerical Analysis I

What are the three basic operations used in Gaussian Elimination with partial pivoting? [3 marks]

Consider the equations
\[
\begin{pmatrix}
5 & 5 & 9 \\
1 & 0.99 & 100 \\
1 & 2 & 3.8
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix}
=
\begin{pmatrix}
0.5 \\
100 \\
2.1
\end{pmatrix}
\]

Perform only the operations described below. Be careful to ensure that results and all intermediate values are rounded to only 2 significant decimal digits. [A calculator may be used, but is not essential.]

(a) Using the first equation as pivot, obtain two equations in \( x_2 \) and \( x_3 \). [4 marks]

(b) Solve the remaining two equations without interchanging equations. Obtain a value for \( x_3 \). [2 marks]

(c) Solve the same two equations again with interchange of equations. Show that the same value of \( x_3 \) is obtained to 2 significant digits. [2 marks]

(d) Use the method of back substitution twice to obtain a pair of solutions \( \{x_1, x_2, x_3\} \) corresponding to steps (b) and (c). [4 marks]

(e) By substituting your results into the original equations, compute vectors of residual errors. Using any suitable norm, determine which of the pair of solutions is more accurate. [5 marks]