Data Structures and Algorithms

Describe and justify Dijkstra’s algorithm for finding the shortest path between two vertices in a directed graph with non-negative lengths associated with its edges.

[8 marks]

For the case where the nodes represent towns and the costs \( C_{uv} \) represent distances by road, Hart, Nilsson & Raphael proposed a variation where the next node to be considered is based on minimising

\[
D(a) + H(a, \text{destination})
\]

instead of the usual \( D(a) \). \( H(u, v) \) is a heuristic function which here should be taken as some constant \( (k, \text{say}) \) multiplied by the Euclidean distance between towns \( u \) and \( v \).

Explain what benefits such a modification might bring and investigate how the correctness and speed of the modified algorithm changes with the value of \( k \).

Can such a variation help in finding the shortest routes to all nodes from a given starting node? Justify your answer.

[12 marks]