

1998 Paper 3 Question 10

Numerical Analysis I

Define *relative error*, *machine epsilon* (*macheps*). [2 marks]

Consider IEEE single-precision arithmetic. How are the 32 bits arranged in terms of sign, exponent and significand? How is the exponent stored? Explain the terms *normalized number*, *denormal number*. What is the *hidden bit* and how is it used? How are negative numbers stored? What does *NaN* stand for? Give an example of an operation that yields a *NaN* value. [6 marks]

Given that $e_{\max} = 127$, show the bit pattern representing each of the following numbers. [Draw lines to separate the sign, exponent and significand. You may use "0...0" to represent long strings of zeros.]

- 0
- $-\infty$
- 1.0
- $1.0 + \text{macheps}$
- 4.0
- $4.0 + \text{macheps}$
- 1.125×2^{-31}
- a *NaN* value [give one example]
- \hat{x} , the smallest representable number greater than 2^{16}

[9 marks]

In the last case, what is

- (a) the value of the least significant bit in the significand of \hat{x} , and
- (b) the relative error if rounding error causes 2^{16} to be stored as \hat{x} ? [3 marks]