1998 Paper 13 Question 13

Numerical Analysis II

Explain what is meant by *local error* and *global error* in methods for the solution of *ordinary differential equations* (*ODEs*). If a typical method has local error $O(h^3)$, what would you expect the global error to be? [3 marks]

Euler's method for solution of $y' = f(x, y)$ can be expressed as $y_{n+1} = y_n + k_1$. From the Taylor series, find an expression for k_1. [2 marks]

The Runge–Kutta method RK2 is

$$y_{n+1} = y_n + \tfrac{1}{2}(k_1 + k_2)$$

where k_1 is the increment used by Euler's method, and

$$k_2 = h\,f(x_n + h, y_n + k_1).$$

In terms of Euler's method, what does the quantity k_2 represent? [2 marks]

Assume that RK2 is carried out with step sizes h and $h/2$, and that

$$y_{(h)}(x_{n+1}) = y(x_{n+1}) + C_n h^2 + O(h^3).$$

Derive an estimate of the error $E_n = |C_n|(h/2)^2$ in $y_{(h/2)}(x_{n+1})$. [3 marks]

Let ε be the *target error per unit step*. Why, in *step-size control* for RK2, is $\varepsilon' = \varepsilon/8$ taken as the target error corresponding to half the step size? [2 marks]

A certain ODE is to be solved using RK2 with step-size control. Using computed values for y from the table below, taking $\varepsilon = 0.005$, and starting with $h = 0.1$, state at which values of x you would make the *first* and *second* changes of step size, and what new values of h you would use in each case.

		h			
		0.025	0.05	0.1	0.2
x	0.05	0.10038	0.10050		
	0.1	0.20279	0.20304	0.20400	
	0.15	0.30946	0.30981		
	0.2	0.42295	0.42341	0.42516	0.43200
	0.25	0.54649	0.54702		
	0.3	0.68434	0.68490	0.68697	
	0.35	0.84247	0.84295		
	0.4	1.02971	1.02989	1.03047	1.03373
	0.45	1.25995	1.25930		
	0.5	1.55646	1.55379	1.54484	

[8 marks]