# COMPUTER SCIENCE TRIPOS Part Ib

Tuesday 2 June 1998 1.30 to 4.30

Paper 4

*Answer* **five** *questions.*
*Submit the answers in five* **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*
*Write on* **one** *side of the paper only.*

## 1 Continuous Mathematics

Consider the family of one-dimensional Gabor wavelets, parameterised for location $x_0$, size $\alpha$, and frequency $\mu_0$. Their functional form is:

$$f(x) = e^{-i\mu_0 x} e^{-(x-x_0)^2/\alpha^2}$$

and their Fourier Transform is:

$$F(\mu) = e^{-ix_0\mu} e^{-(\mu-\mu_0)^2\alpha^2}$$

($a$) Explain the dualities of: ($i$) modulation and shifting; and ($ii$) similarity (reciprocal scaling), in terms of the behaviour of the parameters in the expressions above for $f(x)$ and $F(\mu)$. [4 marks]

($b$) What can you say about the Fourier transform of the *sum* of any two Gabor wavelets? [4 marks]

($c$) What can you say about the Fourier transform of the *product* of any two Gabor wavelets, and why? [4 marks]

($d$) What is the Fourier transform of $f^{(n)}(x)$, the $n^{th}$-derivative of a Gabor wavelet? [4 marks]

($e$) Show that the set of all Gabor wavelets is closed under convolution: *i.e.*, that the convolution of any two Gabor wavelets is itself a single Gabor wavelet. [4 marks]

## 2 Concurrent Systems

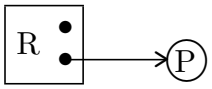
In a system which allocates resources dynamically

($a$) What are the resource allocation policies that make it necessary to consider the possibility of deadlock? [3 marks]

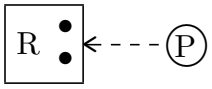
($b$) If there is one instance of each resource type what is the necessary and sufficient condition for deadlock to exist? [2 marks]

Using the notation

- for an instance of a resource




process P has an instance of resource R




process P is requesting an instance of resource R

($c$) Draw a resource–wait graph for five processes where at least three are deadlocked. [5 marks]

($d$) Give the allocation and request matrices corresponding to your graph. [5 marks]

($e$) Illustrate a deadlock detection algorithm using your matrices as an example. [5 marks]

## 3 Programming in Java

Describe briefly the facilities provided in Java for synchronising concurrent threads. [6 marks]

An alternative scheme would be to model the system used in some shops where a machine issues numbered tickets to customers, and customers are served in numeric order. A ticket machine holds an integer, initially zero, and has a single atomic operation:

| | |
|---|---|
| `turn()` | increments the integer and returns its previous value |

A scheduler also holds an integer, initially zero, and has two atomic operations:

| | |
|---|---|
| `next()` | increments the integer count |
| `queue(value)` | suspends the calling thread until the count is at least as large as the value given as an argument |

Given a ticket machine, `m`, and a scheduler, `s`, a critical region could then be coded as follows:

```
number = m.turn();
s.queue(number);
    .
    .       protected code
    .
s.next();
```

Write Java classes `TicketMachine`, with a `turn` method, and `Scheduler`, with `next` and `queue` methods. [8 marks]

Show how a synchronised buffer holding a single value could be implemented using this new scheme. [6 marks]

## 4 Compiler Construction

Sketch parsers based on

(*a*) recursive descent, and [8 marks]

(*b*) a table-driven method of your choice (e.g. SLR(1)) [12 marks]

suitable for parsing the following grammar:

```
S  ->  E eof
E  ->  E + T | E - T | T
T  ->  P ^ T | P
P  ->  ( E ) | n
```

with `S` as the start symbol. The table-driven parser should include the associated algorithm which interprets the table. The parsers do not need to produce a parse tree, merely to report whether the input string is generated by the above grammar. You may assume there is a routine `lex()` which when called places the next symbol (`+`, `-`, `^`, `(`, `)`, `n`, `eof`) in variable `token`.

## 5 Data Structures and Algorithms

Describe and justify Dijkstra's algorithm for finding the shortest path between two vertices in a directed graph with non-negative lengths associated with its edges. [8 marks]

For the case where the nodes represent towns and the costs $C_{uv}$ represent distances by road, Hart, Nilsson & Raphael proposed a variation where the next node to be considered is based on minimising

$$D(a) + H(a, \text{destination})$$

instead of the usual $D(a)$. $H(u, v)$ is a heuristic function which here should be taken as some constant ($k$, say) multiplied by the Euclidean distance between towns $u$ and $v$.

Explain what benefits such a modification might bring and investigate how the correctness and speed of the modified algorithm changes with the value of $k$.

Can such a variation help in finding the shortest routes to all nodes from a given starting node? Justify your answer. [12 marks]

## 6 Computer Design

Briefly describe the differences between a memory bus, an I/O bus and a device interface on a typical PC, with emphasis on latency and bandwidth characteristics. [8 marks]

What is the difference between serial and parallel data transmission? [2 marks]

The physical layer for Universal Serial Bus (USB) is simpler and yet faster than old fashioned RS232. What has made this possible and economic? [5 marks]

Why is it that you can "plug-and-play" with USB devices but you cannot easily "plug-and-play" with RS232 devices? [5 marks]

## 7 Operating System Functions

What is a *translation lookaside buffer* (TLB)? Describe its operation with the aid of a diagram. How is the TLB affected by processor context switches for (*i*) threads and (*ii*) processes? [10 marks]

A process has four page frames allocated to it. (All of the following numbers are decimal, and all numbers start from zero.) The time of the last loading of a page into each page frame, the time of last access to the page, the virtual page number in each frame and the Referenced (R) and Modified (M) bits for each page frame are shown in the table below. Times are in clock ticks from the process start time at time 0.

| Virtual Page # | Frame # | Time Loaded | Time Referenced | M | R |
|---|---|---|---|---|---|
| 2 | 0 | 60 | 161 | 0 | 1 |
| 1 | 1 | 130 | 160 | 0 | 0 |
| 0 | 2 | 26 | 162 | 1 | 0 |
| 3 | 3 | 20 | 163 | 1 | 1 |

A page fault to virtual page 4 has occurred. Which page frame will have its contents replaced under each of the following replacement algorithms? Briefly explain why in each case.

(*a*) FIFO

(*b*) LRU

(*c*) Second Chance (Clock)

(*d*) Enhanced Second Chance

[6 marks]

Given the above state of memory before the fault, and the reference string of virtual page numbers: (4, 0, 0, 0, 2, 4, 2, 1, 0, 3, 2), calculate how many page faults would occur under the LRU policy if a working set with a window size of 4 were used instead of a fixed allocation of 4 frames. Show clearly when each page fault would occur. [4 marks]

## 8 Computation Theory

Explain *Church's Thesis*, making clear its connection with computability. [3 marks]

Define precisely what is meant by the set of all *Primitive Recursive* ($PR$) functions. [4 marks]

Outline steps that would enable you to recursively enumerate the set of all PR functions, showing how to determine the arity of each function generated (little detail is required). [7 marks]

Suppose that $V(n, x)$ is a recursive enumeration of all the PR functions of arity 1. By considering the function $v(x) = S\big(V(x, x)\big)$ or otherwise, show that

($a$) the enumerating function $V(n, x)$ cannot itself be Primitive Recursive; [4 marks]

($b$) there are Total Recursive functions that are not Primitive Recursive. [2 marks]

## 9 Numerical Analysis I

What are the three basic operations used in *Gaussian Elimination with partial pivoting*? [3 marks]

Consider the equations

$$\begin{pmatrix} 5 & 5 & 9 \\ 1 & 0.99 & 100 \\ 1 & 2 & 3.8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 100 \\ 2.1 \end{pmatrix}$$

Perform only the operations described below. Be careful to ensure that results and all intermediate values are rounded to **only 2 significant decimal digits**. [A calculator may be used, but is not essential.]

($a$) Using the first equation as pivot, obtain two equations in $x_2$ and $x_3$. [4 marks]

($b$) Solve the remaining two equations *without* interchanging equations. Obtain a value for $x_3$. [2 marks]

($c$) Solve the same two equations again *with* interchange of equations. Show that the same value of $x_3$ is obtained to 2 significant digits. [2 marks]

($d$) Use the method of *back substitution* twice to obtain a pair of solutions $\{x_1, x_2, x_3\}$ corresponding to steps ($b$) and ($c$). [4 marks]

($e$) By substituting your results into the original equations, compute vectors of residual errors. Using any suitable norm, determine which of the pair of solutions is more accurate. [5 marks]

## 10 Computer Graphics and Image Processing

In ray tracing, *ambient*, *diffuse* and *Phong's specular* shading can be used to define the colour at a point on a surface. Explain what each of the three terms refers to, and what real effect each is trying to model. [3 marks each]

The diagram below represents a scene being ray traced. The circles may be taken to represent the cross-sections of spheres.

In answering the remaining parts of this question you may use the single sheet supplied with the examination paper. Ensure that you attach it to the rest of your answer.

A particular ray from the eyepoint $O$ has been found to have its closest intersection with an object at point $P$. Show, on a diagram, all subsequent rays and vectors which must be found in order to calculate the shading at point $P$. Explain the purpose of each one.

Assume that:

- each object has ambient, diffuse and specular reflections, but is *not* a perfect reflector
- each object is opaque
- all rays and vectors lie in the plane of the paper
- we are *not* using distributed ray tracing

[8 marks]

Assume now that all of the objects are perfect reflectors (in addition to having ambient, diffuse and specular reflection). Show, on a separate diagram, the extra rays which need to be calculated and explain the purpose of each one. [3 marks]