Communicating Automata and Pi Calculus

The main reaction rule of the π-calculus is

\[ \text{COMM} : (M + x(y).P) | (N + \bar{x}(z).Q) \rightarrow \{z/y\}P | Q \]

What other reaction rules are needed to infer all reactions? [4 marks]

Using these rules, show how to infer a reaction on the \( x \)-channel for the following process:

\[ (!x(y).\bar{x}(y)) | (\nu z)(Q | (\bar{x}(z).R + \bar{y}(z).S)) \]

Indicate exactly which rules of structural congruence are required in making the inference. [7 marks]

Let \( P \rightarrow P' \) be an arbitrary reaction inferred for a process \( P \) not containing “+”. Prove, by induction on the depth of the inference, that if \( P \) contains an instance of replication “!” then so does \( P' \). [6 marks]

Show that this is not necessarily true if \( P \) contains “+”. [3 marks]