Types

Let $x$ range over a set of identifiers, $i$ range over integer constants, $r$ range over real constants and $\alpha$ range over a set of type variables. Now suppose we have a set of types, $\sigma$, given by

$$\sigma ::= \alpha \mid \text{int} \mid \text{real} \mid \sigma_1 \times \sigma_2 \mid \sigma_1 \to \sigma_2$$

and a language of terms, $M$, given by

$$M ::= x \mid i \mid r \mid \lambda x. M_1 \mid M_1 M_2 \mid (M_1, M_2).$$

Give ML-like type inference rules for formulae of the form $\Gamma \vdash M : \sigma$, explaining the form and rôle of $\Gamma$. [4 marks]

Explain the notion of principal type and state whether your set of rules has such a property. [4 marks]

Show from your rules that it is impossible to find a $\Gamma$ and $\sigma$ which enable inference of either $\Gamma \vdash \lambda f.(f(1), f(2.7)) : \sigma$ or $\Gamma \vdash \lambda x.xx : \sigma$. [2 marks]

Now suppose we wish to do better than the usual ML treatment of overloading for operators like “+”. So add to the language of types a conjunction connective

$$\sigma ::= \sigma_1 \land \sigma_2$$

where $M : \sigma_1 \land \sigma_2$ means informally that $M$ has both types $\sigma_1$ and $\sigma_2$ and so can be used at either type. Add corresponding inference rules:

$$\frac{\Gamma \vdash M : \sigma \quad \Gamma \vdash M : \sigma'}{\Gamma \vdash M : \sigma \land \sigma'}$$

Now show that a suitable $\sigma_1$ and $\sigma_2$ can be found such that

$$[\text{neg} : (\text{int} \to \text{int}) \land (\text{real} \to \text{real})] \vdash \lambda f.(f(\text{neg}(1)), f(\text{neg}(2.7))) : \sigma_1$$

and

$$[] \vdash \lambda x.xx : \sigma_2$$

hold. [8 marks]

[Hint: for the latter, you might give $x$ a type $\sigma \land \sigma'$ where $\sigma$ is a function type which accepts $\sigma'$ as an argument.]