Numerical Analysis I

Define absolute error and relative error. How are they related? [2 marks]

Consider the quadratic expression \(Q(x) = ax^2 + bx + c\) in which \(a, b, c\) and \(x\) are all represented with the same relative error \(\delta\).

In computing \(bx\), estimate the worst-case relative error, and hence the worst-case absolute error. [2 marks]

Now estimate the worst-case absolute error in computing \(Q(x)\). [4 marks]

Comment on the suitability of the formula

\[
x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}
\]

for computing one of the roots of \(Q(x)\) in floating-point arithmetic. Derive an alternative formula and describe how it could be used in practice. Illustrate your answer by applying it to the case \(a = 4, b = 400, c = 7\) on a decimal machine with only 4 significant digits available. [You may assume \(\sqrt{b^2 - 4ac} \approx b - (2ac/b)\) for the purposes of calculation.] [12 marks]