1997 Paper 1 Question 2

Discrete Mathematics

Suppose set S has m elements and set T has n elements. Give explicit formulae involving m and n only for the functions

$$f(m,n) = |B|/|I| \qquad \text{and} \qquad g(m,n) = |B|/|S \to T|$$

where B and I are the subsets of $S \to T$ consisting of those functions which are respectively bijections and injections.

[Hint: for f it may help to consider for each subset $X \subseteq T$, the number of injections which have range exactly X and then count the number of such X.] [10 marks]