Continuous Mathematics

Suppose that some continuous function $f(x)$ has Fourier Transform $F(\mu)$. Now consider the consequences in the Fourier domain of each of the following operations upon $f(x)$:

(a) What will be the Fourier Transform of the $n$th derivative of $f(x)$ with respect to $x$: $\frac{d^n}{dx^n} f(x)$? [4 marks]

(b) What will be the Fourier Transform after shifting $f(x)$ by a distance $\alpha$: $f(x - \alpha)$? [4 marks]

(c) What will be the Fourier Transform after dilating $f(x)$ by a factor $\alpha$: $f(x/\alpha)$? [4 marks]

(d) Suppose that $f(x)$ is convolved with another function $g(x)$ whose Fourier Transform is $G(\mu)$. What will be the Fourier Transform $H(\mu)$ of the convolution result $h(x)$ where

$$h(x) = \int_{-\infty}^{+\infty} f(\alpha)g(x - \alpha)d\alpha?$$

[4 marks]

(e) Suppose now that a two-dimensional continuous function $f(x,y)$ has a 2D Fourier Transform $F(\mu,\nu)$. Define the Laplacian operator $\nabla^2$ and express the 2D Fourier Transform of $\nabla^2 f(x,y)$. [4 marks]