COMPUTER SCIENCE TRIPOS Part IB

Monday 2 June 1997 1.30 to 4.30

Paper 3

Answer **five** *questions.*
Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*
Write on **one** *side of the paper only.*

1 Continuous Mathematics

Suppose that some continuous function $f(x)$ has Fourier Transform $F(\mu)$. Now consider the consequences in the Fourier domain of each of the following operations upon $f(x)$:

(a) What will be the Fourier Transform of the nth derivative of $f(x)$ with respect to x: $\dfrac{d^n}{dx^n} f(x)$? [4 marks]

(b) What will be the Fourier Transform after shifting $f(x)$ by a distance α: $f(x - \alpha)$? [4 marks]

(c) What will be the Fourier Transform after dilating $f(x)$ by a factor α: $f(x/\alpha)$? [4 marks]

(d) Suppose that $f(x)$ is convolved with another function $g(x)$ whose Fourier Transform is $G(\mu)$. What will be the Fourier Transform $H(\mu)$ of the convolution result $h(x)$ where

$$h(x) = \int_{-\infty}^{+\infty} f(\alpha)g(x - \alpha)d\alpha?$$

[4 marks]

(e) Suppose now that a two-dimensional continuous function $f(x, y)$ has a 2D Fourier Transform $F(\mu, \nu)$. Define the Laplacian operator ∇^2 and express the 2D Fourier Transform of $\nabla^2 f(x, y)$. [4 marks]

[TURN OVER

2 Concurrent Systems

In a given application, processes may be identified as "readers" or "writers". Multiple-reader, single-writer access to data is required to be implemented with priority for writers. Readers execute procedures `startread` and `endread` before and after reading. Writers execute `startwrite` and `endwrite`.

Give pseudocode for the `startread` procedure using

(*a*) semaphores

(*b*) conditional critical regions

(*c*) monitors

(*d*) guarded commands

Explain via comments in your pseudocode how concurrency control is achieved in each case and criticise each approach from the viewpoint of the programmer and the implementer.

[20 marks]

3 Further Modula-3

The thread system in Modula-3 uses mutexes and condition variables to control concurrency. An alternative scheme would be to use message passing.

Each message-passing channel would be an object with two methods:

```
send (message: REFANY)
```

and

```
receive (): REFANY
```

plus an initialisation method.

Calling `send` would append the argument to a (fixed-length) FIFO queue of data which would be collected by the thread calling `receive`. The thread would block if `receive` were called when the queue was empty or if `send` were called when the buffer was full.

Write an interface, `Message`, defining a suitable opaque object type with `init` and `send` methods, and write a further interface `MessageInternal` revealing the `receive` method. [8 marks]

Write an implementation exporting to both of these interfaces supplying the concrete revelations of the types and providing appropriate default methods.

[12 marks]

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4 Compiler Construction

Consider the following grammar for expressions (`<E>`) and commands (`<C>`).

```
<E> ::= i | n | <E> - <E> | <E> ** <E> | ( <E> )

<C> ::= i := <E>
      | if <E> then <C> | if <E> then <C> else <C>
      | <C> repeatwhile <E> | <C> ; <C> | { <C> }
```

Show that there are syntactic ambiguities between (a) the minus (`-`) and exponentiation (`**`) operators, (b) the `if`-command and the `if-then-else`-command, and (c) the `if-then-else`-command and the `repeatwhile`-command.

[4 marks]

Define, in a programming language notation of your choice, a recursive descent parser that will construct the abstract syntax tree for an input stream conforming to the above syntax for commands. You may assume the existence of a function `lex()` that will yield an integer representing the next lexical token from the input stream, and the functions `mk2(op,x)`, `mk3(op,x,y)` and `mk4(op,x,y,z)` that will construct abstract syntax tree nodes with a given operator and one, two or three operands. You should assume that exponentiation is right associative and more binding than subtraction which is left associative. The command following `then` should be the longest possible and the command before `repeatwhile` should be the shortest possible.

[12 marks]

Briefly outline how you would modify your parser if the command to the left of `repeatwhile` was changed to be the longest (rather than the shortest). [4 marks]

5 Data Structures and Algorithms

You are given two sets of numbers X_i and Y_j, where i and j run from 1 to N.

Devise an algorithm to find the M largest values of $X_i - Y_j$. This algorithm should not be quadratic in N, though it is permitted to be quadratic in M.

You should regard N as being of the order of 20,000 and M as being of the order of 1,000.

[20 marks]

6   Data Structures and Algorithms

Describe and justify Kruskal's algorithm for finding the minimum spanning tree of an undirected graph.   [6 marks]

Suppose that all edges longer than some given L were omitted from the graph, for example as a result of not calculating them at all. Would the algorithm still give the correct result? Would you be able to tell if it had not? If it yielded a tree would it be guaranteed to be the best one? Justify your answers and consider the problem of finding a minimum spanning tree for 1,000,000 points in a plane rectangle where there is an edge between every pair of points and the cost of the edge is the Euclidean distance between the two points.   [14 marks]

7   Structured Hardware Design

A *smart card* is the size and shape of a credit card, but contains a microprocessor, a RAM, a ROM and some amount of non-volatile storage. The only connections to the card are five gold-plated contacts for power, ground, clock, data and reset. Write a few sentences about each of the following:

(*a*) Design partition: i.e. how should the circuitry of the card be structured?

[5 marks]

(*b*) Testing: i.e. how will the component(s) of the card and the completed card be tested before shipping?   [5 marks]

(*c*) Product customisation: i.e. how will the card be different in different application markets (banking, authentication, casino, cellular phone etc.)?

[5 marks]

(*d*) Uniqueness: i.e. how would the card be customised or differentiated on a per-card basis for individual users?   [5 marks]

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8 Operating System Functions

A computer is equipped with a CPU with a 32-bit virtual address space, a 32-entry TLB with access time 10ns and 32 Mbyte DRAM with access time 100ns. Its secondary storage is provided by an IDE hard disc with transfer rate 1 Mbyte/s, and which rotates at 3600 revolutions per minute and has an average seek time of 10ms. The computer uses demand paged virtual memory with 1 kbyte pages.

(*a*) Explain the function of the TLB with the aid of a diagram.   [4 marks]

(*b*) Design a page table structure which the operating system can use to implement virtual memory on this system and describe how a virtual address is translated using it. Are there any drawbacks of your approach?   [4 marks]

(*c*) What is *demand paging*? Briefly describe a policy which the operating system can use to share the available physical memory between competing processes.
[3 marks]

(*d*) Calculate the effective memory access time for the system if virtual memory is managed using the scheme described in your answer to (*b*), if page tables are kept locked in memory, the probability of finding a translation in the TLB is 98%, and the probability of a page fault is 10^{-6}. Exclude operating system overhead from your calculation, and briefly explain your answer.   [9 marks]

9 Computation Theory

Explain the action of a Turing machine, and show how the progress of a computation may be tracked by maintaining a record of the configuration at each time t. Prove that a computation which enters the same configuration twice will not terminate.
[8 marks]

Suppose you are given a Turing machine T having r states and k symbols. It is known that in a particular computation the head moves on the tape so that it is never more than l squares from its starting point. Calculate a bound on the number of configurations that the machine may enter during the computation.   [4 marks]

State a precise form of the unsolvability of the HALTing problem for Turing machines. Assuming this result, show that it is not possible to compute a bound on the distance of the head from its starting position during HALTing Turing machine computations.   [8 marks]

[You may assume that a Turing machine computation may be characterised by codes q and d that specify the quintuplet description and initial tape contents uniquely.]

10 Numerical Analysis I

Define *absolute error* and *relative error*. How are they related? [2 marks]

Consider the quadratic expression $Q(x) = ax^2 + bx + c$ in which a, b, c and x are all represented with the same relative error δ.

In computing bx, estimate the worst-case relative error, and hence the worst-case absolute error. [2 marks]

Now estimate the worst-case absolute error in computing $Q(x)$. [4 marks]

Comment on the suitability of the formula

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

for computing one of the roots of $Q(x)$ in floating-point arithmetic. Derive an alternative formula and describe how it could be used in practice. Illustrate your answer by applying it to the case $a = 4$, $b = 400$, $c = 7$ on a decimal machine with only 4 significant digits available. [You may assume $\sqrt{b^2 - 4ac} \simeq b - (2ac/b)$ for the purposes of calculation.] [12 marks]