Answer **two** questions from Section A, and **one** question from each of Sections B, C, D and E.

Submit the answers in **six separate** bundles, each with its own cover sheet. On each cover sheet, write the numbers of **all** attempted questions, and circle the number of the question attached.

Write on **one** side of the paper only.

## SECTION A

### 1 Foundations of Computer Science

The datatype PRI defined below is to be used for the representation of priority queues which are finite or infinite ordered sets of integers.

```
datatype PRI = E
  | N of int*(unit->PRI);
```

Define an ML function \( \text{intfromto}(i,j) : (\text{int}*\text{int})\rightarrow\text{PRI} \) which will return a representation of the ordered set of integers \( \{ i, i+1, \ldots, j \} \).

Define the function \( \text{first}(p) : \text{PRI}\rightarrow\text{int} \) that will return the first (and hence smallest) integer in the given queue \( p \), and \( \text{rest}(p) : \text{PRI}\rightarrow\text{PRI} \) that will return (if possible) a representation of the given queue \( p \) with its smallest element removed. Your implementation should be such that the expression

\[
\text{first}(\text{rest } (\text{intfromto}(20, 1000000)))
\]

should evaluate efficiently.

Define an ML function \( \text{ins}(i,p) : (\text{int} \times \text{PRI})\rightarrow\text{PRI} \) which will return a priority queue with the integer \( i \) inserted in the proper position of the given queue \( p \).
2 Discrete Mathematics

Suppose set $S$ has $m$ elements and set $T$ has $n$ elements. Give explicit formulae involving $m$ and $n$ only for the functions

\[ f(m, n) = |B|/|I| \quad \text{and} \quad g(m, n) = |B|/|S \rightarrow T| \]

where $B$ and $I$ are the subsets of $S \rightarrow T$ consisting of those functions which are respectively bijections and injections.

[Hint: for $f$ it may help to consider for each subset $X \subseteq T$, the number of injections which have range exactly $X$ and then count the number of such $X$.] [10 marks]

3 Programming in Modula-3

Consider the following program:

```modula-3
MODULE Squares EXPORTS Main;

IMPORT IO;

PROCEDURE Square (i: INTEGER): INTEGER =
BEGIN
  RETURN i * i;
END Square;

BEGIN
  FOR i := 1 TO 10 DO
    IO.Put (Fmt.Int (i) & ^2 = & Fmt.Int (Square (i)) & "\n);
  END;
END Squares.
```

Rewrite the program to use an exception to return the result of the function. [10 marks]

4 Operating Systems

Describe the various functions involved in interrupt handling. Indicate the hardware and software that might be involved in their implementation. [7 marks]

Discuss the interaction of interrupt-driven software and process scheduling in an operating system. [3 marks]
SECTION B

5 Foundations of Computer Science

Noughts and Crosses is a game played by two players (O and X) on a board with nine positions numbered as follows:

```
  1  2  3
  4  5  6
  7  8  9
```

The players place their marks (O and X) in unoccupied positions on the board until the game is complete. A completed game is when either

(a) there is a straight line of three Xs giving a win for X, or

(b) there is a straight line of three Os giving a win for O, or

(c) all nine positions are occupied, in which case the game is drawn.

O is the first player to move.

It is required to construct an ML structure representing the tree of all possible games. Each node of the tree should represent a reachable board state, with the root being the empty board, and the leaf nodes corresponding to won, lost or drawn games.

Define the ML datatype `tree` that you would use to represent this game tree.

Define the function `mktree : unit->tree` to construct the complete game tree, explaining carefully how it works. There is no need for your implementation to be efficient in either space or time.

Briefly discuss ways in which your implementation of `mktree` could be made more efficient.

Define a function `Owins : tree->int` which when applied to the complete tree will yield the number of distinct games in which O wins.
6 Foundations of Computer Science

A rooted directed graph has vertices identified by integers. Each vertex \( v \) has a left successor given by \( \text{left}(v) \) and a right successor given by \( \text{right}(v) \), where \( \text{left} \) and \( \text{right} \) are ML functions of type \( \text{int} \rightarrow \text{int} \). The graph contains the root and all vertices reachable by paths from the root.

Define a datatype \( G \) that could be used to represent such a graph with given root, and left and right functions, and define a function \( \text{mkgraph}(\text{root}, \text{left}, \text{right}) \) that can create values of type \( G \). Show that such values can be used to represent both finite and infinite graphs. [4 marks]

A path through the graph is represented by a \( \text{bool} \) list with \( \text{true} \) and \( \text{false} \) indicating left and right edges, respectively.

Define the function \( \text{last} : G \rightarrow \text{bool list} \rightarrow \text{int} \) that will yield, for a given graph, the identity of the vertex reached by following the given path from the root. [3 marks]

In a new application, where \( \text{last} \) is repeatedly called, it is required for it to return both the identity of the last vertex and a count of how often this particular vertex has been returned. Define a new version of the datatype \( G \), containing mutable values, that could be used. [3 marks]

Illustrate the use of this datatype by defining the new versions of \( \text{mkgraph} \) and \( \text{last} \). [10 marks]
SECTION C

7 Discrete Mathematics

Let us say that a finite partial order \((A, \sqsubseteq)\) is tree-like if, for every \(a \in A\), the set (of its predecessors) \(\{x \in A \mid x \sqsubseteq a \land x \neq a\}\) either is empty or has a unique maximal element. Equivalently, pictorially, this holds when the Hasse diagram of \(A\) consists of one or more trees.

State which of the following relations on the integers \(\{1, 2, \ldots, 10\}\) are tree-like partial orders and give a one-sentence justification.

(a) \(R\) where \(xRy \iff x = y\)

(b) \(R\) where \(xRy \iff x \leq y\) (here \(\leq\) is the usual ordering on integers)

(c) \(R\) where \(xRy \iff x\) divides-exactly-into \(y\)

(d) \(R\) where \(xRy \iff x = y\) or \(x\) is the greatest prime factor of \(y\)

[8 marks]

To count the number \(C(n)\) of tree-like partial orders of \(n\) elements, assume \(A = \{1, 2, \ldots, n\}\) and then place each element \(i\) in turn into a Hasse diagram starting from 1 and such that no later element \(j > i\) is placed such that \(j \sqsubseteq i\).

Show that, provided \(n > 1\), we have \(C(n) = f(n, C(n - 1))\) and give the function \(f(n, m)\). Provide a base case and thereby solve the recurrence for \(C(n)\). [12 marks]
8 Discrete Mathematics

Let \( A \) be a set and \( R \) a relation on \( A \); also write \( R^k \) for the usual \( k \)-fold composition of \( R \), i.e. \( R^1 = R, R^{k+1} = R \circ R^k \). Let \( t(R) \) be the smallest relation which is transitive and has \( R \subseteq t(R) \), similarly let \( u(R) = \bigcup_{k=1}^{\infty} R^k \).

(a) Show (e.g. by induction on \( k \)) that \( (\forall k \geq 1) R^k \subseteq t(R) \);

(b) deduce \( u(R) \subseteq t(R) \).

(c) Show further that \( u(R) \) is transitive;

(d) and hence argue that \( u(R) = t(R) \). [8 marks]

Now suppose \( A \) has \( n \) elements; give a sketch of an algorithm which takes as input an \( n \times n \) boolean array \( V \) representing \( R \) above and produces as output a similar boolean array \( W \) representing \( t(R) \) above. Give the running time of your algorithm in the form \( O(f(n)) \). [8 marks]

Find a set \( A \) and a relation \( R \) on \( A \) which is not reflexive, but where \( t(R) \) is reflexive. [4 marks]

SECTION D

9 Programming in Modula-3

Describe the facilities in Modula-3 for controlling the repeated execution of a block of code. [10 marks]

Show how FOR, REPEAT and WHILE loops can be emulated by using LOOP and EXIT. [10 marks]

10 Programming in Modula-3

Describe the facilities in Modula-3 for modelling data by means of ARRAYs, RECORDs and REFerences. [12 marks]

Illustrate your answer by specifying some data types to model a map. This should include points (specifying latitude and longitude), junctions (specifying a point and the roads incident at the junction) and road segments (specifying the junctions at each end, a name and some intermediate points between the ends). [8 marks]
SECTION E

11 Operating Systems

Why does the widespread use of graphical user interfaces (GUIs) make explicit the need for the underlying operating system to support concurrent processes and threads? [2 marks]

Outline the data structures that might be held by an operating system to support the entities that are scheduled (processes or threads). [8 marks]

Describe one scheduling algorithm and explain how it would be implemented, based on the data structures you have described above. [7 marks]

What are the implications for scheduling of the need to support new media types such as voice and video? [3 marks]

12 Operating Systems

What information would you expect a filing system to keep on a file? Discuss where this information might be held. [4 marks]

Criticise each of the following methods of recording the disc blocks allocated to a file. In each case, indicate the suitability of the method for new media types such as voice and video.

(a) chaining in the media

(b) detached chain

(c) table of pointers

(d) extents [16 marks]