Additional Topics

Explain the concept of *environment* as used in Natural Semantics. In particular, explain its use in the delivery judgements which describe the *evaluation* of phrases in Standard ML. What are the main kinds of *result* which such an evaluation may deliver, for different kinds of phrase? [6 marks]

The following Standard ML program has three top-level phrases:

```ml
val step = 5;
fun sum(0) = 0 | sum(n) = step*n + sum(n-1);
sum(3);
```

What are the components of the three environments in which these three phrases are evaluated? During the evaluation of `sum(3)`, the expression `sum(n-1)` is evaluated several times; in which environments do these evaluations take place? [7 marks]

Give two examples of theorems which one may be able to prove about the delivery judgements for evaluation, and explain the practical relevance of these theorems. [7 marks]