Logic and Proof

Briefly contrast the Davis–Putnam proof procedure with resolution. Illustrate your answer using proofs using both methods of

\[(P \to R) \land (\neg P \to \neg Q) \land (P \lor Q) \to (P \land R)\]  

[8 marks]

A polynomial over the integers, using modulo-2 arithmetic, can be regarded as a Boolean formula under the correspondence 1 \(\mapsto\) \textbf{true} and 0 \(\mapsto\) \textbf{false}. Show how to translate an arbitrary propositional formula to an equivalent polynomial, describing the translations of \(\neg A\), \(A \land B\), \(A \lor B\), \(A \to B\) and \(A \leftrightarrow B\).  

[5 marks]

Use this translation to show that \((A \land B) \leftrightarrow (B \land A)\) is a tautology.  

[2 marks]

Use this translation to give a rule for simplifying formulæ of the form

\[(\ldots ((A \leftrightarrow A) \leftrightarrow A) \ldots \leftrightarrow A)\]  

[5 marks]