

1996 Paper 4 Question 9

Numerical Analysis I

Let x^* be the floating-point representation of a number x . Define the *absolute error* and *relative error* in representing x by x^* . How are these errors related? [3 marks]

Let x_1, x_2 be two numbers. Find expressions for

(a) the *absolute error* in representing $x_1 + x_2$

(b) the *relative error* in representing $x_1.x_2$ (where “.” denotes multiplication) [4 marks]

Assume that the numbers 1 and 2 are represented exactly. Find an expression for the absolute error in calculating $2x + 1$. [2 marks]

In an iterative calculation the number y is an improved value of x , derived from the assignments

$$p := x/2 + 1$$

$$q := x - 2$$

$$y := p + 1/q$$

If ε_x is the absolute error in representing x , find an expression for the *absolute error* ε_y in representing y . [6 marks]

What is the approximate *relative error* δ_y in representing y when $x = 2.01$? [5 marks]