

## 1996 Paper 4 Question 8

### Computation Theory

A *bag*  $B$  of natural numbers is a total function  $f_B : \mathbb{N} \rightarrow \mathbb{N}$  giving for each natural number  $x$  the count  $f_B(x)$  of occurrences of  $x$  in  $B$ . If each  $f_B(x) = 0$  or  $1$ , then  $f_B$  is the characteristic function  $\chi_S$  of a set  $S$ : every set can thus be regarded as a bag.

- (a) A bag  $B$  is *recursive* if the function  $f_B$  is computable. Suppose that the sequence of bags  $\{B_n \mid n \in \mathbb{N}\}$  is recursively enumerated by the computable function  $e(n, x) = f_n(x)$ , which gives the count of  $x$  in each bag  $B_n$ . Show that there is a recursive set  $S$  that is different from each bag  $B_n$ . [7 marks]

Hence prove that the set of all recursive bags cannot be recursively enumerated. [3 marks]

- (b) A bag  $B$  is *finite* if there is  $X \in \mathbb{N}$  such that  $f_B(x) = 0$  for all  $x \geq X$ . Show that the set of all finite bags is recursively enumerable. [10 marks]