

1996 Paper 2 Question 5

Probability

If n coins are tossed, the number of ways in which r can land heads is ${}^n C_r$. Given that $n, r \in \mathbb{N}$ and that $0 \leq r \leq n$, Pascal's Theorem states:

$${}^n C_r = \begin{cases} 1, & \text{if } r = 0 \text{ or } r = n \\ {}^{n-1} C_{r-1} + {}^{n-1} C_r, & \text{otherwise} \end{cases}$$

Prove Pascal's Theorem.

[6 marks]

Hence prove that:

$${}^n C_r = \frac{n!}{(n-r)!r!}$$

[7 marks]

Show that:

$$\sum_{r=0}^n {}^n C_r = 2^n$$

[7 marks]