Probability

If \( n \) coins are tossed, the number of ways in which \( r \) can land heads is \( \binom{n}{r} \). Given that \( n, r \in \mathbb{N} \) and that \( 0 \leq r \leq n \), Pascal’s Theorem states:

\[
\binom{n}{r} = \begin{cases} 
1, & \text{if } r = 0 \text{ or } r = n \\
\binom{n-1}{r-1} + \binom{n-1}{r}, & \text{otherwise}
\end{cases}
\]

Prove Pascal’s Theorem. \[6 \text{ marks}\]

Hence prove that:

\[
\binom{n}{r} = \frac{n!}{(n-r)!r!}
\]

[7 marks]

Show that:

\[
\sum_{r=0}^{n} \binom{n}{r} = 2^n
\]

[7 marks]