

1996 Paper 1 Question 7

Discrete Mathematics

State the requirements for (S, \leq) to be a partially-ordered set. Define the notion of a *topological sort* of S . [10 marks]

Let (S, \leq_S) and (T, \leq_T) be partially ordered sets. We say that $f : S \rightarrow T$ is an *isomorphism* of (S, \leq_S) and (T, \leq_T) if f is a bijection and it preserves the order i.e.

$$(\forall x, y \in S)(x \leq_S y \Leftrightarrow f(x) \leq_T f(y)).$$

If such an isomorphism exists we say that (S, \leq_S) and (T, \leq_T) are *isomorphic*.

Give two topological sorts of $\mathbb{N} \times \mathbb{N}$ ordered by the product order, one of which is isomorphic to \mathbb{N} with the usual ordering and the other is not. [10 marks]