Discrete Mathematics

Show that the number of undirected bi-partite graphs which have \( n \) vertices is

\[
\sum_{k=0}^{n} \binom{n}{k} 2^{k(n-k)}.
\]

For the purposes of this question regard such an undirected bi-partite graph as a triple \((V, W, E)\) having disjoint sets of vertices \(V\) and \(W\) (with \(|V \cup W| = n\)) and edges \(E \subseteq (V \times W)\). Note this means that the two graphs \((\{1, 2\}, \{3\}, \{\}\)) and \((\{1\}, \{2, 3\}, \{\})\) are counted separately (because their partition differs) whereas one would more commonly argue that they are the same graph.

[10 marks]