Mathematics for Computation Theory

Let $E$ be an event over $S$ that is accepted by the deterministic finite automaton $M \equiv (Q, S, \iota, f, A)$, where $|Q| = N$. Suppose that $z \in E$ is a word such that $\ell(z) \geq N$: show that we may write $z =uvw$ where

(i) $\ell(uv) \leq N$

(ii) $\ell(v) \geq 1$

(iii) for all $n \geq 0$, $uv^n w \in E$ [12 marks]

State whether each of the following languages over $S = \{a, b\}$ is regular, giving your reasons.

(a) $L_1 = \{ww \mid w \in S^*\}$ [6 marks]

(b) $L_2 = \{wzw \mid w, z \in S^*\}$ [2 marks]

[Note: $|Q|$ indicates the number of elements in set $Q$, and $\ell(w)$ the number of characters in word $w$.]