COMPUTER SCIENCE TRIPOS Part II

Wednesday 5 June 1996 1.30 to 4.30

Paper 8

Answer **five** *questions.*
Submit the answers in five **separate** *bundles each with its own cover sheet.*
Write on **one** *side of the paper only.*

1 Specification and Verification I

Describe how a theory of program refinement can be defined on top of Floyd–Hoare logic. [6 marks]

Define the specification notation $[P, Q]$. [4 marks]

Show that:

```
[X=n ∧ n>0,  Z=n!]
⊇
BEGIN
  VAR Y;
    Y := 1;
    Z := Y;
    WHILE Y < X DO (Y := Y+1; Z := Z×Y)
END
```

[10 marks]

2 Digital Communication II

Discuss the techniques which an efficient transport layer protocol might use to determine when to (re)transmit a packet. [10 marks]

Under what circumstances might such a protocol not make full use of an available window at a receiver? What effect would this have on remote logins and file transfer connections? [5 marks]

The memory buffer capacity of a certain bottlenecked router in the Internet is much less than one packet per connection traversing it. What effects will this have on TCP? [5 marks]

3 Computer System Modelling

A telephone exchange multiplexes 64 Kb/s voice calls onto a 256 Kb/s trunk line (therefore the line will hold at most four calls). New calls have an exponentially distributed inter-arrival process, with a mean of 20 seconds, and the call holding time is exponentially distributed with a mean of 60 seconds.

(*a*) Draw a diagram of a Markov Chain which models the system, labelling the state transitions with their rates where appropriate. What is the necessary condition for stability of this system? [5 marks]

(*b*) Derive an expression for the probability that an arriving call finds k calls in progress, for $k \geqslant 0$, and thence calculate the probability that a caller finds the exchange engaged, given the parameters above. [15 marks]

4 Comparative Architectures

Discuss implementing the ANSI C routine `memcpy()` in hardware and software. Why might a C compiler place a string literal on a word boundary when it only needs to be byte aligned? [10 marks]

The DEC ALPHA is a RISC-inspired instruction set designed to last around 25 years with multiple implementations as hardware. Discuss to what extent the epithet RISC can be applied and to what extent its main architectural features follow the familiar RISC mould. [10 marks]

5 Business Studies

In a project plan what is meant by a *critical path*, and why is the concept useful? [5 marks]

A certain software project has two phases. Each phase has three tasks (analysis, coding and testing) which must be performed sequentially. Analysis for phase 2 cannot be started until analysis for phase 1 is complete. The effort in person-weeks, as estimated by the programmers, in each task is given in the table below.

Task	**Phase 1**	**Phase 2**
Analysis	4	8
Coding	4	4
Testing	4	8

Two staff are assigned to the project. Tasks may be performed by either member of staff. Analysis and coding tasks can have only one person usefully working at a time whereas, for testing, the time to completion is inversely proportional to the effort expended.

Draw PERT and GANTT charts for the project, and indicate the critical path. [5 marks]

Staff are each paid £24,000 per annum. The company allows for overheads of 100% of salary. Equipment worth £50,000 will be needed, depreciated over 5 years. Payment is proposed as 25% on start, 25% on delivery of the first phase, and 50% on completion. Draw up a rough budget and cash flow for the project. How much working capital (excluding equipment purchase, but including depreciation) is required? [5 marks]

What price and delivery would you quote for this project? Explain the additional factors you have considered in formulating your quotation. [5 marks]

6 Advanced Algorithms

(a) Sketch a *Binomial Heap* data structure containing the four values $\{2, 4, 6, 8\}$.

(b) Sketch a Binomial Heap storing the thirteen values $\{1, 3, \ldots, 25\}$.

Note that in parts (a) and (b) the values can be arranged in the heaps in several different ways, while still satisfying all the conditions required of a Binomial Heap. Your heaps will of course store the smallest values at the top, but you should explain how much flexibility there was beyond that and what policy you adopted in placing values. [10 marks]

(c) Form the union of the above two heaps, explaining the steps used and showing where the stored values end up. You do not need to display *all* the pointers in your data structures, and need not include any elaborate discussions of other operations on or applications of binomial trees or heaps. [10 marks]

7 Optimising Compilers

Explain what is meant by *register allocation by colouring* from the point of view of a compiler-writer (do not give theory, but explain principal concepts and sketch central algorithms). You may assume that the input is in three-address instruction form, but state any presumed properties on temporaries; moreover indicate carefully what form the output from the register allocation phase takes, particularly occurrences of user-variables. [8 marks]

For a machine with n registers give a program which results in not all the (non-address taken) user-variables being allocatable to registers. [2 marks]

Consider compiling the following source file of a program

```
extern void p(void), q(int,int);
extern int f(int);
void proc()
{   int a,b,c;
    a = f(1), b = f(2); p(); q(a,b);
    b = f(3), c = f(4); p(); q(b,c);
    c = f(5), a = f(6); p(); q(c,a);
}
```

on a machine which has eight registers preserved over function call and sufficient temporaries and argument registers corrupted over function call available for allocation. Clearly the code can be compiled using three registers, one for `a`, `b` and `c`. However, it is obvious to any assembly language programmer how two registers suffice. Suggest how your answer to the first part of this question could be adapted to compile the above code better.

[Hint: the usual phrase for this concept is *variable splitting according to live ranges*. You may wish to consider general flowgraphs after considering basic blocks.] [6 marks]

Why might the above idea be helpful in general? Would it affect the resulting code speed, resulting code size or the compilation time? Would it improve the final machine code for `proc` on a register-based architecture of your choice? [4 marks]

8 Artificial Intelligence

Discuss the automatic discovery of heuristics for problem solving. Take into account the questions of "credit assignment", and the use of "relaxed" versions of the problem. Illustrate your discussion with examples from at least two problems. [20 marks]

9 Database Topics

Describe briefly the traditional relational model of data, explaining what is meant by *First Normal Form* (1NF). [4 marks]

Comment on the strengths and weaknesses of the model. [6 marks]

In what ways does the Object Data Model proposed in the standard ODMG-93 surmount the difficulties that you have identified? [10 marks]

10 Designing Interactive Applications

Some of today's photocopiers are connected by networks to repair centres so that technicians can monitor their performance and detect problems without visiting customer premises. Although this offers cost savings, it can have a negative impact on customer relations. Suggest an explanation for this, drawing on your knowledge of the service technician's job. [5 marks]

You have been asked to design a modification to a networked photocopier, to enable users to send messages to the repair centre when they encounter problems. Again drawing on your understanding of the nature of photocopier repair work, produce a rough design for the message-system user interface. Include a one-sentence problem statement, a mental-model description and an outline of the design of the user interface itself. [15 marks]

11 Information Theory and Coding

Let X and Y represent random variables with associated probability distributions $p(x)$ and $p(y)$, respectively. They are not independent. Their conditional probability distributions are $p(x|y)$ and $p(y|x)$, and their joint probability distribution is $p(x, y)$.

(a) What is the *marginal entropy* $H(X)$ of variable X, and what is the *mutual information* of X with itself? [4 marks]

(b) In terms of the probability distributions, what are the *conditional entropies* $H(X|Y)$ and $H(Y|X)$? [4 marks]

(c) What is the *joint entropy* $H(X, Y)$, and what would it be if the random variables X and Y were independent? [4 marks]

(d) Give an alternative expression for $H(Y) - H(Y|X)$ in terms of the joint entropy and both marginal entropies. [4 marks]

(e) What is the *mutual information* $I(X; Y)$? [4 marks]

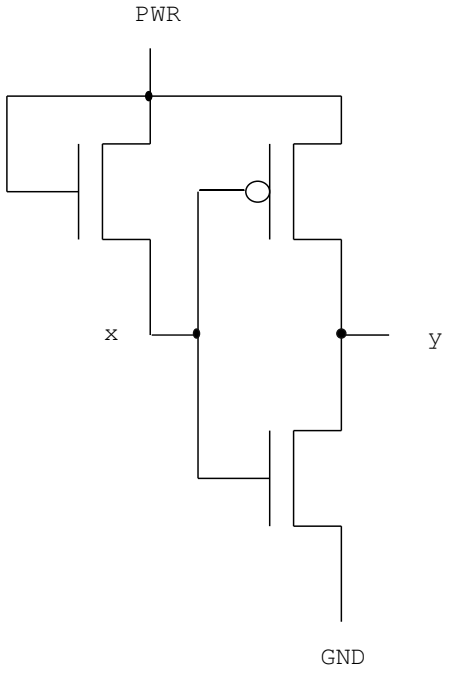
12 Computer Vision

Discuss the problem of face recognition and face detection based on wavelet encodings of facial structure and facial features. How can one distinguish between those facial undulations that are generic (universal, or normally present in all faces), and those which are particular to a given face and which therefore distinguish it from others? How can statistical decision theory formalize these two pattern recognition problems – face detection and face recognition? What are the main advantages and disadvantages of using wavelets for the encoding of faces? [20 marks]

13 Specification and Verification II

Discuss the difficulties in finding a model of transistors that is suitable for formal verification. [10 marks]

Consider the circuit:



What can be deduced about the values at `x` and `y`

(*a*) using the simple switch model? [5 marks]

(*b*) using the difference switching model? [5 marks]

14 Numerical Analysis II

In Peano's theorem, if a quadrature rule integrates polynomials of degree N exactly over an interval $[a, b]$, then the error in integrating $f \in C^{N+1}[a, b]$ is conventionally expressed as

$$E(f) = \int_a^b f^{(N+1)}(t) K(t)\, dt$$

where

$$K(t) = \frac{1}{N!} E_x[(x-t)_+^N].$$

Explain the notation $(x-t)_+^N$ and E_x. [3 marks]

It follows directly from Taylor's theorem that

$$E(f) = \frac{1}{N!} E_x \left\{ \int_a^x f^{(N+1)}(t)(x-t)^N\, dt \right\}.$$

Explain clearly, in simple stages, how to complete the proof of Peano's theorem. [8 marks]

For the mid-point rule, what is N? [1 mark]

If $K(t)$ does not change sign in $[a, b]$ then

$$E(f) = \frac{f^{(N+1)}(\xi)}{(N+1)!} E(x^{N+1})$$

for some $\xi \in (a, b)$. Use this result to simplify

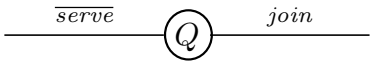
$$E(f) = \int_{-1}^{1} f(x)\, dx - 2f(0)$$ [8 marks]

15 Pi Calculus

What are the three kinds of *commitment* for a process in the π calculus? Explain informally how the commitments of $P \mid Q$ arise from those of P or Q or both.

[6 marks]

We wish to implement a queue, to be weakly equivalent to the specification



$$Q\langle\rangle \stackrel{\text{def}}{=} join(x).Q\langle x\rangle$$
$$Q\langle x_1 \ldots x_n\rangle \stackrel{\text{def}}{=} join(x).Q\langle x_1 \ldots x_n x\rangle + \overline{serve}\langle x_1\rangle.Q\langle x_2 \ldots x_n\rangle$$

The body of the queue is to consist of a chain of cells, each having the form $\overline{a}\langle x, b\rangle$; to hold a queue of n items $x_1 \ldots x_n$ we define a chain parametrically on its head a_0 and tail a_n as follows:

$$Cells\langle x_1 \ldots x_n\rangle \stackrel{\text{def}}{=} (a_0 a_n)(\nu a_1 \ldots a_{n-1})\big(\overline{a}_0\langle x_1, a_1\rangle \mid \cdots \mid \overline{a}_{n-1}\langle x_n, a_n\rangle\big)$$

(a) Define agents $Server\langle a\rangle$ and $Joiner\langle b\rangle$ to manage the head and tail of the queue, respectively, with the intention that the system $Queue\langle x_1 \ldots x_n\rangle$ defined below should be weakly equivalent to $Q\langle x_1 \ldots x_n\rangle$:

$$Queue\langle\rangle \stackrel{\text{def}}{=} (\nu a)\big(Server\langle a\rangle \mid Joiner\langle a\rangle\big)$$
$$Queue\langle x_1 \ldots x_n\rangle \stackrel{\text{def}}{=} (\nu ab)\big(Server\langle a\rangle \mid Cells\langle x_1 \ldots x_n\rangle\langle ab\rangle \mid Joiner\langle b\rangle\big)$$

[7 marks]

(b) In terms of your definition of $Server$ and $Joiner$, work out the commitments of $Queue\langle x_1 \ldots x_n\rangle$ far enough to give an informal argument that it is indeed weakly equivalent to $Q\langle x_1 \ldots x_n\rangle$.

[Hint: treat the cases $n = 0$ and $n \neq 0$ separately.] [7 marks]