Logic and Proof

Briefly describe propositional logic, first-order logic and modal logic. Outline the syntax and semantics of each. [5 marks]

Describe the main features of the sequent calculus. (It is not necessary to give the rules for particular connectives.) [2 marks]

Explain informally why the following sequent calculus rules are sound, adding provisos if necessary:

\[
\frac{A, B, \Gamma \Rightarrow \Delta}{A \land B, \Gamma \Rightarrow \Delta} \quad \frac{A, \Gamma \Rightarrow \Delta}{\exists x A, \Gamma \Rightarrow \Delta}
\] [4 marks]

Prove \( \exists x (A \land B) \Rightarrow (\exists x A) \land B \) using the sequent calculus. Add any necessary provisos and explain why they are necessary. [9 marks]