1995 Paper 6 Question 12

Semantics

The language IMP$'$ comprises integer and boolean expressions and commands, defined by

$$\begin{array}{rcrl} ie & \in & \mathit{Iexp} & ::= \underline{n} \mid x \mid ie_1\ \underline{iop}\ ie_2 \\ be & \in & \mathit{Bexp} & ::= \underline{b} \mid ie_1\ \underline{bop}\ ie_2 \\ C & \in & \mathit{Com} & ::= \mathtt{skip} \mid x := ie \mid (C_1; C_2) \mid \\ & & & \ \mathtt{if}\ be\ \mathtt{then}\ C_1\ \mathtt{else}\ C_2 \mid \mathtt{repeat}\ C\ \mathtt{until}\ be \end{array}$$

where $n \in \mathbb{Z}$, $b \in \{\mathit{true}, \mathit{false}\}$, $\mathit{iop} \in \{+, \times, -\}$, $\mathit{bop} \in \{<, =\}$ and $x \in \mathit{Pvar}$, a set of program variables.

(a) Give an annotated evaluation semantics for IMP$'$, expressing the usual behaviour of each command and expression form, which derives statements of the forms

$$ie, S \Rightarrow_I n; R \qquad be, S \Rightarrow_B b; R \qquad C, S \Rightarrow_C S'; R, W$$

for $S, S' \in \mathit{States} = (\mathit{Pvar} \to \mathbb{Z})$ and $R, W \subseteq \mathit{Pvar}$. $C, S \Rightarrow_C S'; R, W$ means 'in state S, command C executes to state S' whilst reading the set of variables R and writing the set of variables W'. Similarly, if $e \in \mathit{Iexp} \cup \mathit{Bexp}$ then $e, S \Rightarrow v; R$ means 'in state S, e reads variables R in evaluating to v'.

[5 marks]

(b) For $be \in \mathit{Bexp}$, $ie \in \mathit{Iexp}$, $C \in \mathit{Com}$ use induction on the structure of phrases to give simple definitions of sets

$$\mathcal{R}(ie), \mathcal{R}(be), \mathcal{PR}(C), \mathcal{PW}(C) \subseteq \mathit{Pvar}$$

where $\mathcal{R}(e)$ is the set of variables accessed by e, $\mathcal{PR}(C)$ is a set of variables possibly read during the execution of C and $\mathcal{PW}(C)$ is a set of variables possibly written to during the execution of C. Give an example to show that it is *not* in general true that

$$C, S \Rightarrow_C S'; R, W \quad \text{implies} \quad W = \mathcal{PW}(C).$$

[5 marks]

(c) Prove that for any C, S, S', R, W

$$C, S \Rightarrow_C S'; R, W \quad \text{implies} \quad (\forall x \in \mathit{Pvar}.\ x \notin W \Rightarrow S(x) = S'(x))$$

and that for any be, S, S', b, R

$$(\forall x \in \mathcal{R}(be). S(x) = S'(x)) \quad \text{implies} \quad (be, S \Rightarrow_B b; R \iff be, S' \Rightarrow_B b; R)$$

[5 marks]

(d) Prove that for any C_1, C_2, C_3 and be that if $\mathcal{R}(be) \cap \mathcal{PW}(C_1) = \emptyset$ then

$$(C_1; \mathtt{if}\ be\ \mathtt{then}\ C_2\ \mathtt{else}\ C_3) \approx \mathtt{if}\ be\ \mathtt{then}\ (C_1; C_2)\ \mathtt{else}\ (C_1; C_3)$$

You may assume without proof that if $C, S \Rightarrow_C S'; R, W$ then $W \subseteq \mathcal{PW}(C)$.

[5 marks]