1995 Paper 5 Question 10

Foundations of Functional Programming

Let \( A \equiv \lambda x \ y. \ (x \ x \ y) \)
\( \Theta \equiv A \ A \)
\( suc \equiv \lambda n \ f. \ f \ (n \ f \ x) \)
\( true \equiv \lambda x \ y. \ x \)
\( false \equiv \lambda x \ y. \ y \)

Reduce each of the following \( \lambda \)-terms to normal form (if possible) and to head normal form (hnf) (if possible).

\[
\begin{align*}
\Theta \ suc & \quad \Theta \ (suc \ x) \\
\Theta \ true & \quad \Theta \ false \\
\Theta \ (\lambda x. \ x \ x) & \quad \Theta \ (\lambda x. \ f \ x \ x)
\end{align*}
\]

[12 marks]

If \( M \) has no hnf then \( M[N/x] \) has no hnf, for all \( N \). Use this fact to show the following:

If \( M \) has no hnf then \( M \ N \) has no hnf, for all \( N \). [8 marks]