Numerical Analysis II

With reference to solution of the differential equation \( y' = f(x, y) \), explain the conventional notation \( x_n, y(x_n), y_n, f_n \). [4 marks]

Derive Euler’s method

\[ y_{n+1} = y_n + hf(x_n, y_n). \] \hspace{1cm} (1) [3 marks]

Euler’s method has local error \( \frac{h^2}{2} y''(\xi) \).

Explain the terms local error, global error. [2 marks]

The multistep formula

\[ y_{n+1} = y_{n-3} + \frac{4h}{3} (2f_n - f_{n-1} + 2f_{n-2}) \] \hspace{1cm} (2)

has local error \( \frac{14}{45} h^5 y^{(5)}(\xi) \).

Outline the technique for deriving multistep formulae such as (2). (Omit algebraic details.) [2 marks]

Suppose Euler’s formula is used as a starting procedure for formula (2). How many initial steps of formula (2) need to be evaluated using Euler? [2 marks]

Estimate very roughly the number \( N \) of Euler steps needed to approximate \( f_1 \). (Assume that \(| y^{(5)}(x) | \approx 30 \), and Euler’s method has global error \( h/N \).) [5 marks]

What is the most important requirement for a starting procedure? Suggest a more suitable starting procedure than Euler. [2 marks]