Semantics of Programming Languages

Let $D$ be a complete partial order with bottom. What does it mean for a subset of $D$ (regarded as a predicate) to be inclusive? State Scott’s principle of fixed-point induction. [6 marks]

Let $B$ be the usual flat complete partial order of truth values consisting of elements $\perp$, $true$ and $false$. For a complete partial order $D$ with bottom, let the conditional function

$$
\cdot \rightarrow | \cdot : B \times D \times D \rightarrow D
$$

be given by

$$
b \rightarrow d | e = \begin{cases} 
d & \text{if } b = true, 
\ e & \text{if } b = false, 
\perp & \text{if } b = \perp.
\end{cases}
$$

Let $p : D \rightarrow B$ and $h : D \rightarrow D$ be continuous functions with $h$ strict (i.e. $h(\perp) = \perp$). Let $f : D \times D \rightarrow D$ be the least continuous function which satisfies

$$
f(x, y) = p(x) \rightarrow y | h(f(h(x), y)) \quad \text{for all } (x, y) \in D \times D.
$$

Show that the following predicate

$$
P(g) \equiv \forall (x, y) \in D \times D. h(g(x, y)) = g(x, h(y))
$$

is inclusive. [4 marks]

Prove that $h(f(x, y)) = f(x, h(y))$, for all $(x, y) \in D \times D$. [10 marks]