In $2n$ independent trials the probability of success is $p_1$ in each of the first $n$ trials and $p_2$ in the remaining $n$. Prove that the mean and variance of the total number of successes are $n(p_1 + p_2)$ and $n(p_1^2 + p_2^2)$ respectively. [10 marks]

Hence show that unless $p_1 = p_2$ the variance of the number of successes is less than it would be in a binomial distribution with the same number of trials and the same mean number of successes. [10 marks]