1994 Paper 10 Question 7

Data Structures and Algorithms

For the following, n is a positive integer and G is a graph of N nodes (vertices) and E arcs (edges) each with a given weight (or cost). For *seven* of the following indicate, with a short justification, whether the statement is true or false.

(a) All functions f of the form $f(n) = An^k$ (with A and k being constants) are in the class $O(2^n)$.

(b) All sorting methods for an array of n elements take time $O(n^5)$.

(c) It is possible to sort an array of n elements using binary comparisons in $\Theta(n \log n)$ time.

(d) It is possible to sort an array of n elements using binary comparisons using $O(1)$ (i.e. constant independent of n) additional space.

(e) Radix sorting can sort any set of integers in linear time.

(f) All straight lines from the inside of a polygon to the outside intersect the points on the edges forming its boundary an odd number of times.

(g) It is always cheaper to find the shortest distance between two given nodes u, v of G than to find all N shortest distances from u to every other node.

(h) It is possible to find the shortest paths between all N^2 pairs of nodes of G in $O(N^3)$ time.

(i) If G is connected then the minimal spanning subtree of G contains the $N - 1$ edges whose weights are smallest.

(j) Given n points $(x_i, y_i), 1 \leqslant i \leqslant n$ in a plane, then the four points (x_a, y_a), (x_b, y_b), (x_c, y_c), (x_d, y_d) such that x_a is minimal of the x_i, x_b is maximal of the x_i, y_c is minimal of the y_i, y_d is maximal of the y_i form a quadrilateral Q which can be used to speed up a convex hull algorithm by preprocessing to remove points which lie inside Q.

Marks will be awarded for overall succinctness, attention to detail and absence of random guesses lacking justification.

[20 marks]