

## 1993 Paper 9 Question 10

### Semantics

Explain what is a *well-founded* binary relation, and state the principle of well-founded induction. [3 marks]

Show that the binary relation  $\triangleleft$  on the integers which is given by

$$m \triangleleft n \quad \text{if and only if} \quad n < m \leq 100$$

is well-founded. [2 marks]

Consider the ML declarations

```
fun f(x) = if x > 100 then (x - 10) else f(f(x + 11));
fun g(x) = if x > 100 then (x - 10) else 91;
```

Prove, by induction on the well-founded relation  $\triangleleft$ , that  $f$  and  $g$  determine equal integer-valued functions.

Hint: for the induction step you may find it helpful to consider separately the cases  $x > 100$ ,  $x = 100$ ,  $90 \leq x < 100$  and  $x < 90$ . [15 marks]