Semantics

Explain what is a well-founded binary relation, and state the principle of well-founded induction. [3 marks]

Show that the binary relation $\triangleright$ on the integers which is given by

$$m \triangleright n \quad \text{if and only if} \quad n < m \leq 100$$

is well-founded. [2 marks]

Consider the ML declarations

```ml
fun f(x) = if x > 100 then (x - 10) else f(f(x + 11));
fun g(x) = if x > 100 then (x - 10) else 91;
```

Prove, by induction on the well-founded relation $\triangleright$, that $f$ and $g$ determine equal integer-valued functions.

Hint: for the induction step you may find it helpful to consider separately the cases $x > 100$, $x = 100$, $90 \leq x < 100$ and $x < 90$. [15 marks]