

1993 Paper 8 Question 1

Digital Signal Processing

Show that an N -point Discrete Fourier Transform (DFT)

$$X(p) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi np/N}$$

may be evaluated in terms of two $\frac{N}{2}$ -point DFTs if N is even. [6 marks]

Without giving a detailed mathematical derivation, discuss how this result may be used to give the Fast Fourier Transform algorithm. Discuss the advantages of the algorithm compared with direct evaluation of the DFT. [5 marks]

Discuss briefly the use of window functions in discrete spectrum analysis. [3 marks]

The generalised Hamming window function is defined by

$$\begin{aligned} w(n) &= \alpha - (1 - \alpha) \cos(2\pi n/N) && \text{for } 0 \leq n < N \\ &= 0 && \text{otherwise} \end{aligned}$$

$$\text{where } 0 \leq \alpha \leq 1.$$

Obtain an expression for the DFT of this window function. [6 marks]